# Composable Core-sets for Determinant Maximization: A Simple Near-Optimal Algorithm


Piotr Indyk
MIT

**Sepideh Mahabadi**
TTIC

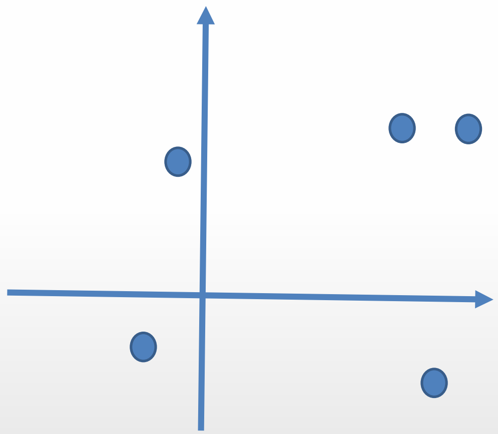
Shayan Oveis Gharan
U. of Washington

Alireza Rezaei
U. of Washington

# Volume (Determinant) Maximization Problem

**Input:** a set of $n$ vectors $\mathrm{V} \in \mathbb{R}^d$ and a parameter $k \leq d$,

$\boldsymbol{k = 2}$



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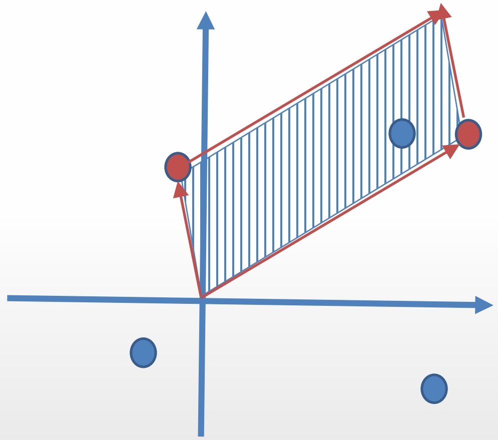

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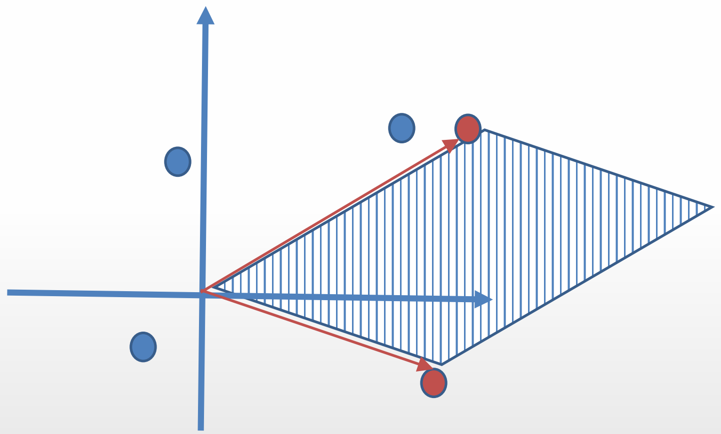

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$$\begin{bmatrix} v_1 \; v_2 \dots v_n \end{bmatrix}$$

$$\boldsymbol{V}$$

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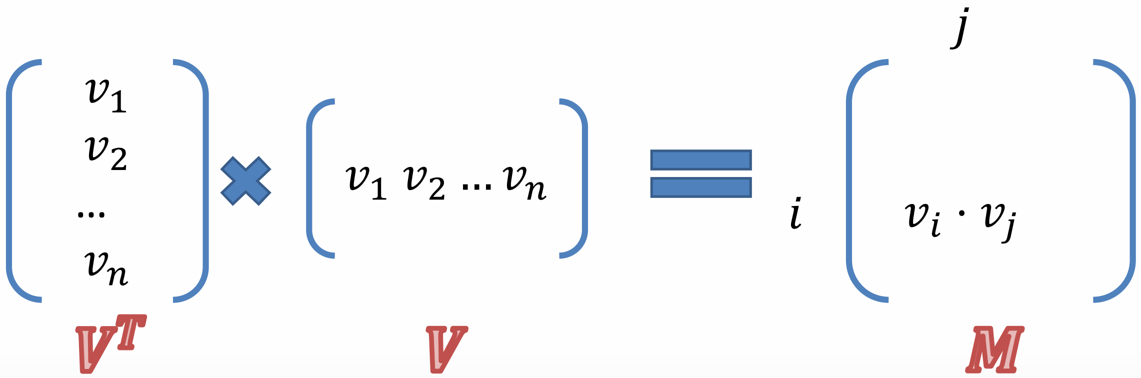


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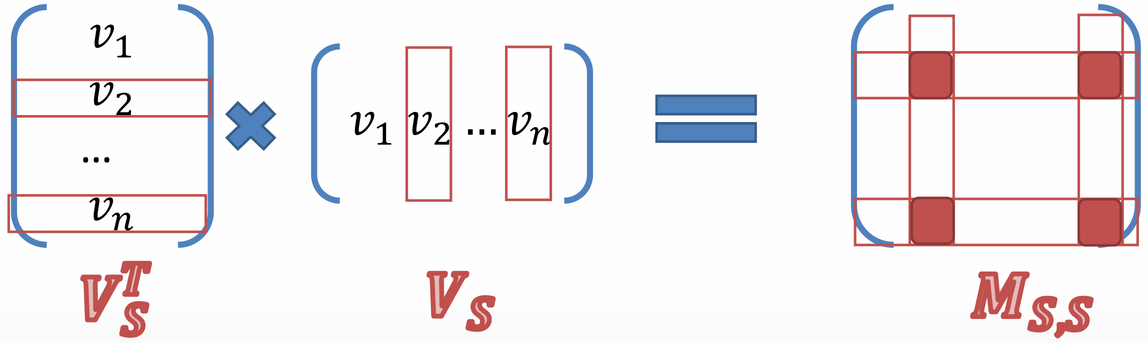


**Equivalent Formulation:**

Reuse $V$ to denote the matrix where its columns are the vectors in $V$

- Let $M$ be the gram matrix $V^T V$
- Choose $S$ such that $\det(M_{S,S})$ is maximized

$M_{i,j} = v_i \cdot v_j$

$\det(M_{S,S}) = Vol(S)^2$

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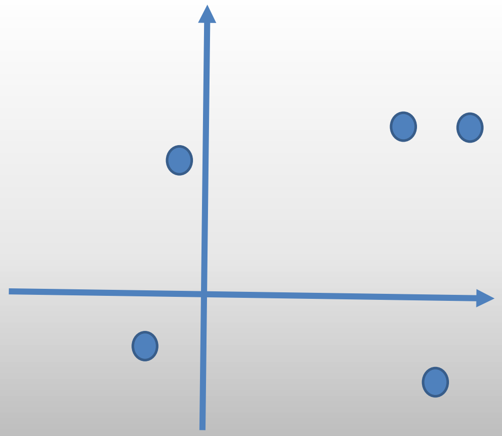

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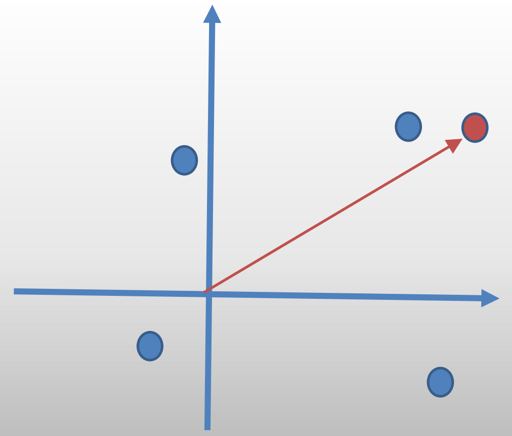

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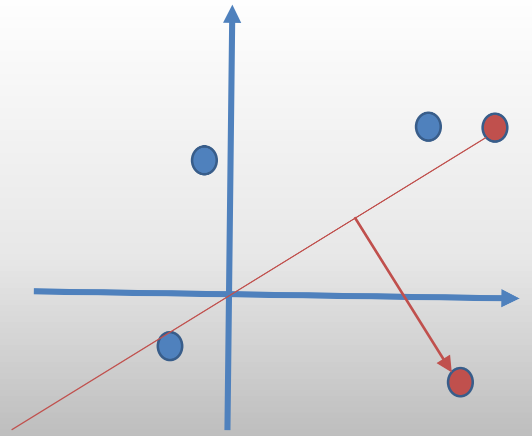

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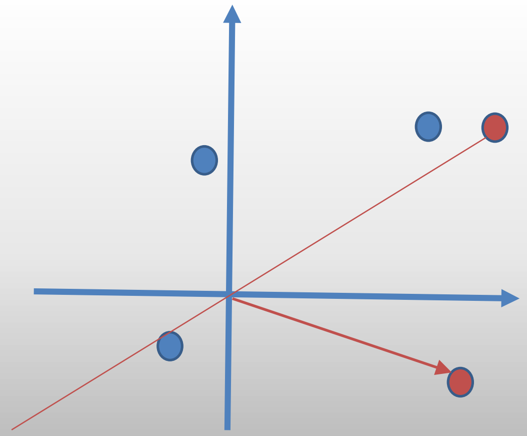

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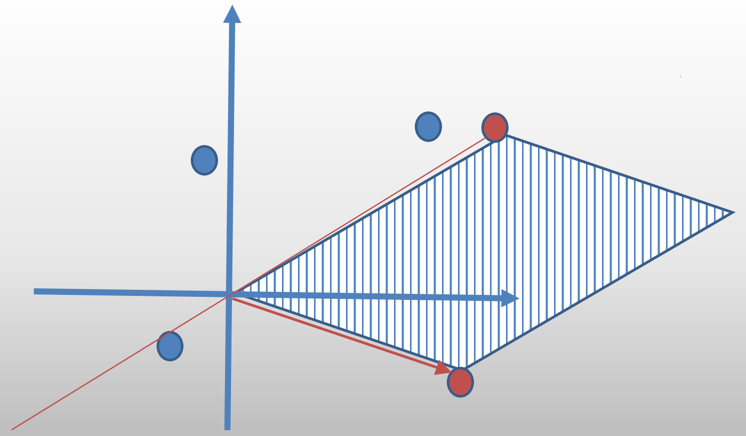

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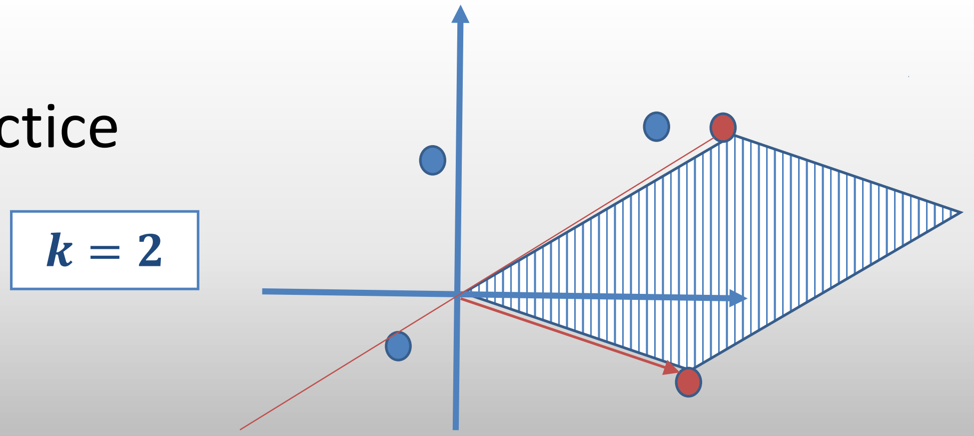
- Greedy performs very well in practice



# Determinantal Point Processes (DPP)

**DPP:** Very popular probabilistic model, where given a set of vectors $V$, **samples** any $k$-subset $S$ with probability proportional to this determinant.

- Maximum a posteriori (MAP) decoding is determinant maximization
- Volume/determinant is a notion of **diversity**

- **NeurIPS'18 Tutorial,** *Negative Dependence, Stable Polynomials, and All That*, Jegelka, Sra
- **ICML'19 Workshop,** *Negative Dependence: Theory and Applications in Machine Learning,* Gartrell, Gillenwater, Kulesza, Mariet

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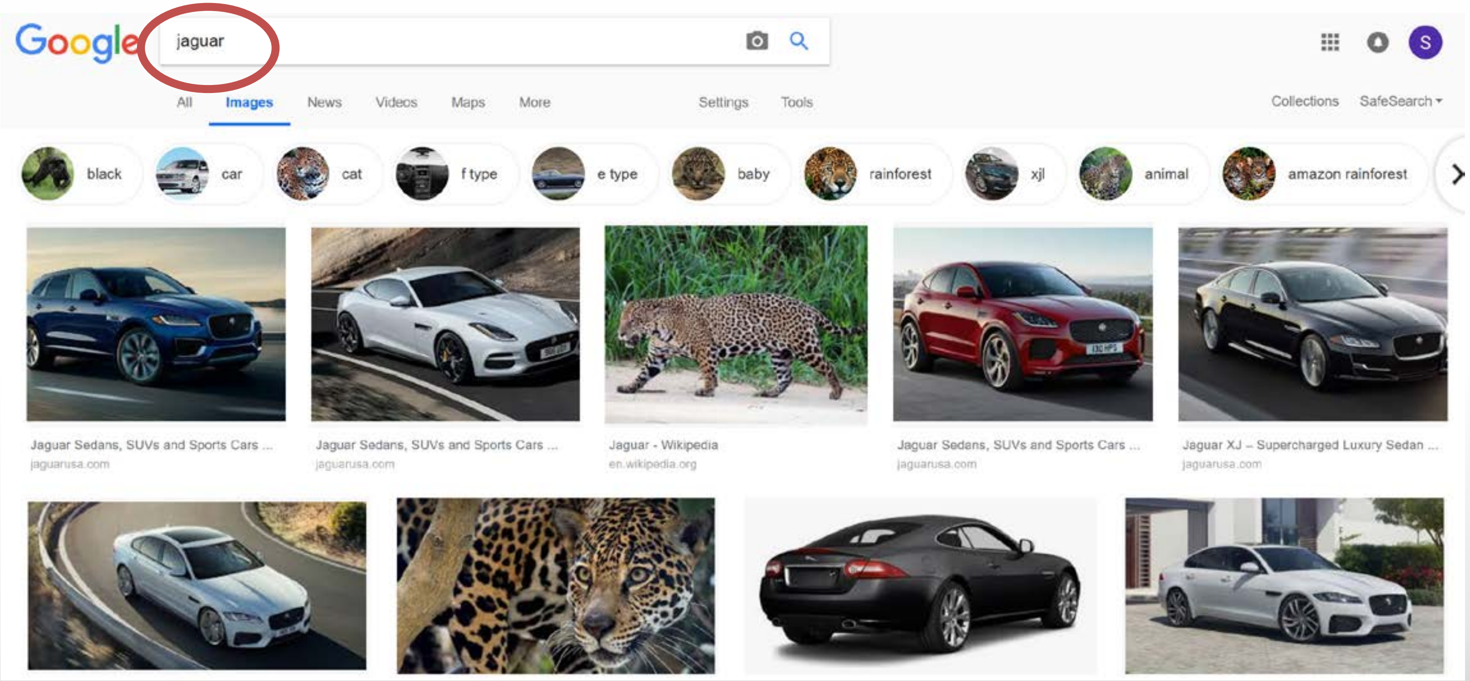

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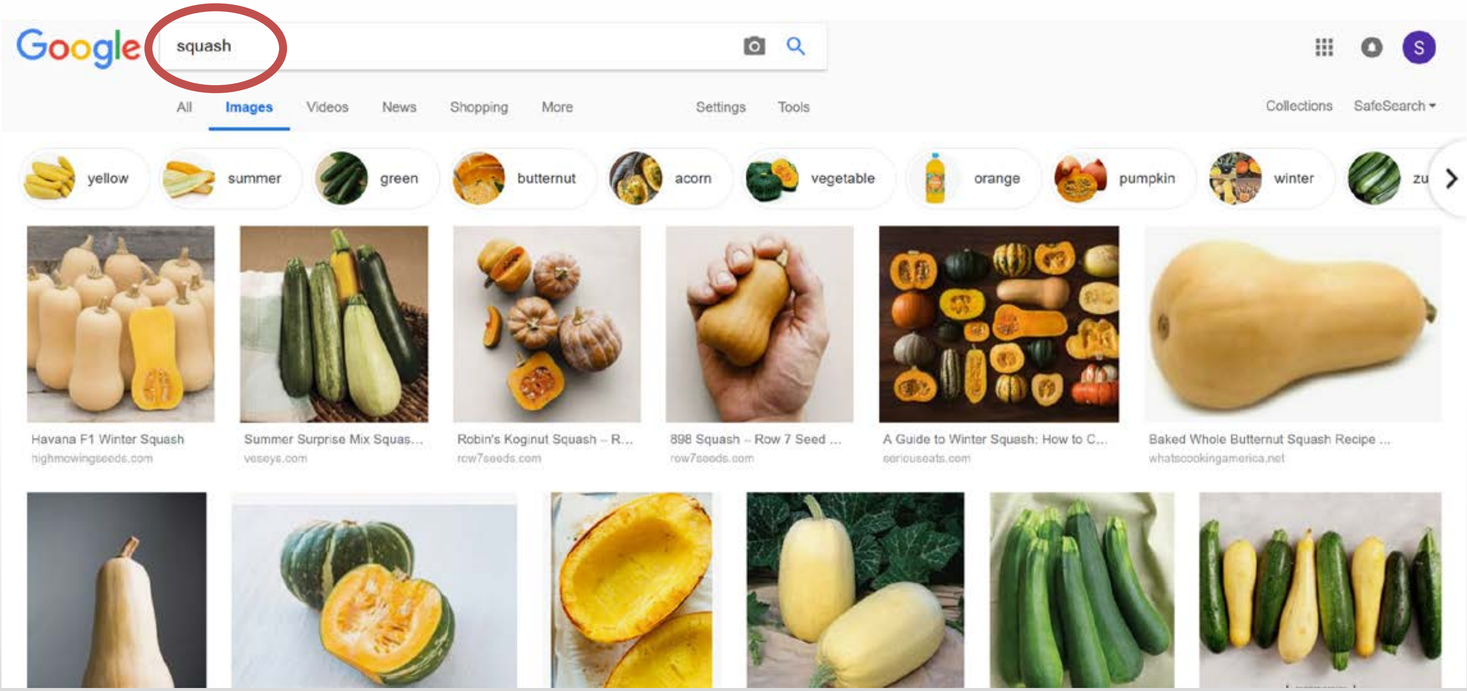
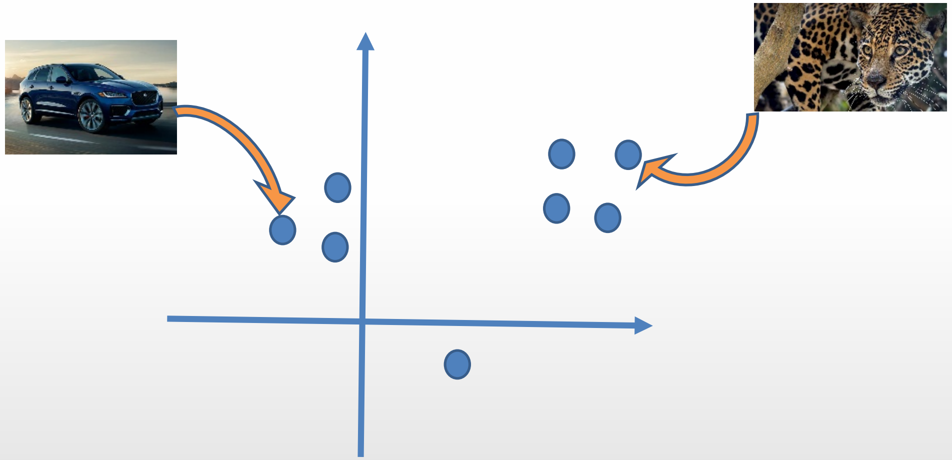


# Application: Diversity Maximization

**Objects (documents, images, etc)** → Feature Vectors → **Points in a high dimensional space**



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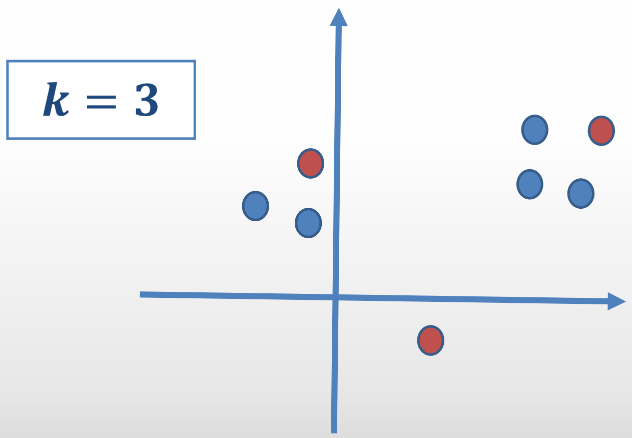
**Goal:** pick $k$ points while maximizing "diversity".



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- Lots of effort for solving the problem in massive data models of computation [MJK'17, WIB'14, PJG+'14, MKSK'13, MKBK'15, MZ'15, MZ'15, BENW'15]
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**Composable Core-sets**

# Core-sets

**Core-sets** [AHV'05]**:** a subset $U$ of the data $V$ that represents it well

Solving the problem over $U$ gives a good approximation of solving the problem over $V$

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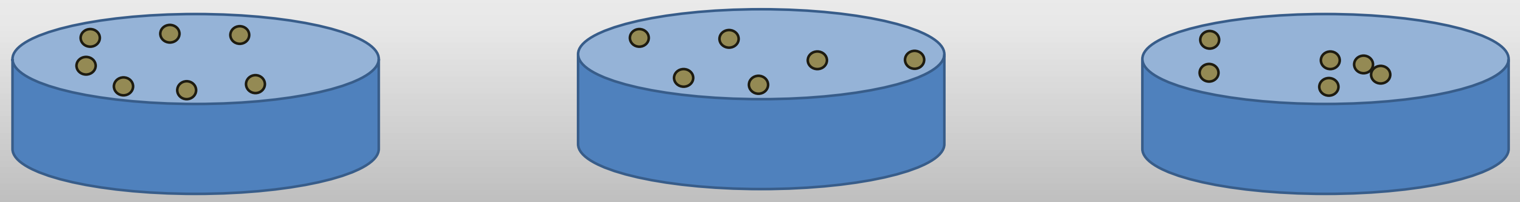

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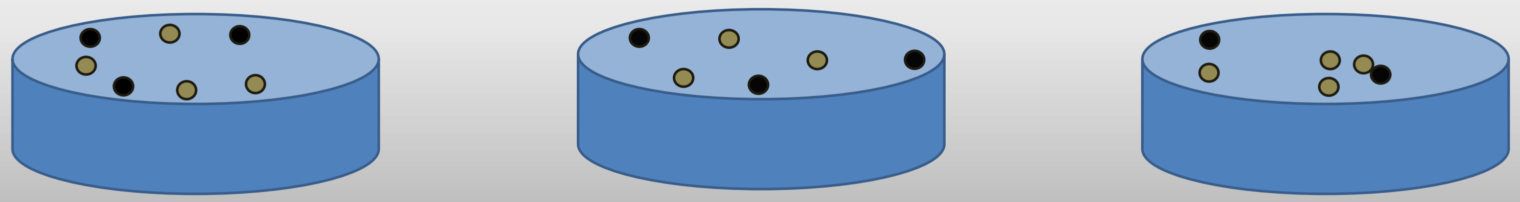

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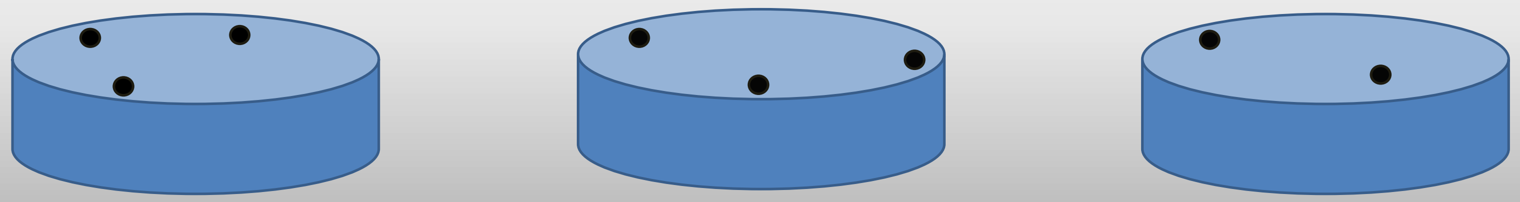

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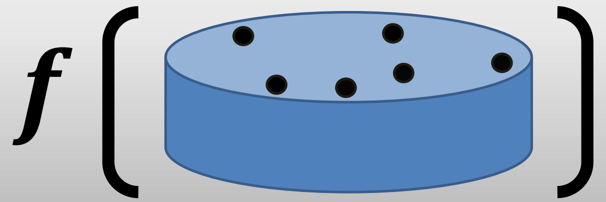

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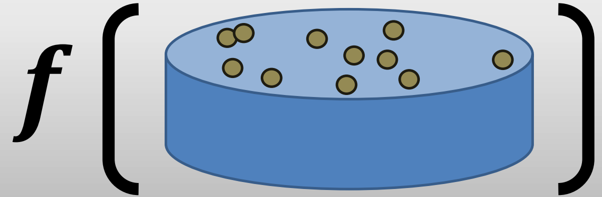

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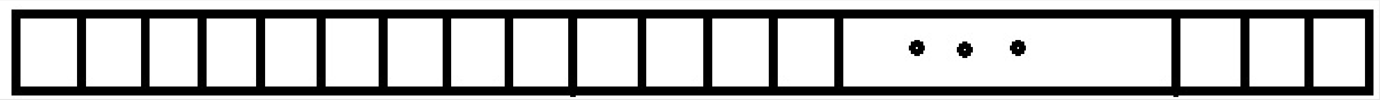
✓ Composable Core-sets have been studied for the **diversity Maximization** problems, for other notions of diversity: minimum pairwise distance, sum of pairwise distances, etc.

✓ Determinant maximization is a "higher order" notion of diversity

# Applications: Streaming Computation

- **Streaming Computation:**
  - Processing sequence of $n$ data elements “on the fly”
  - limited Storage



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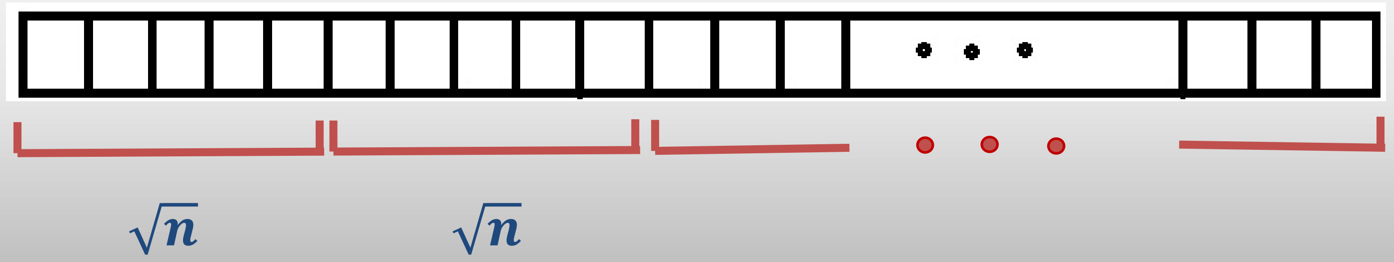

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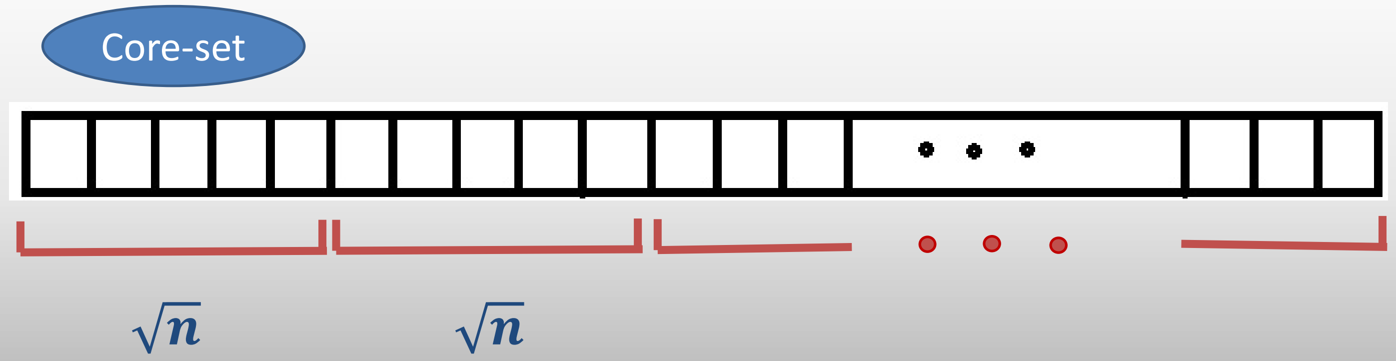

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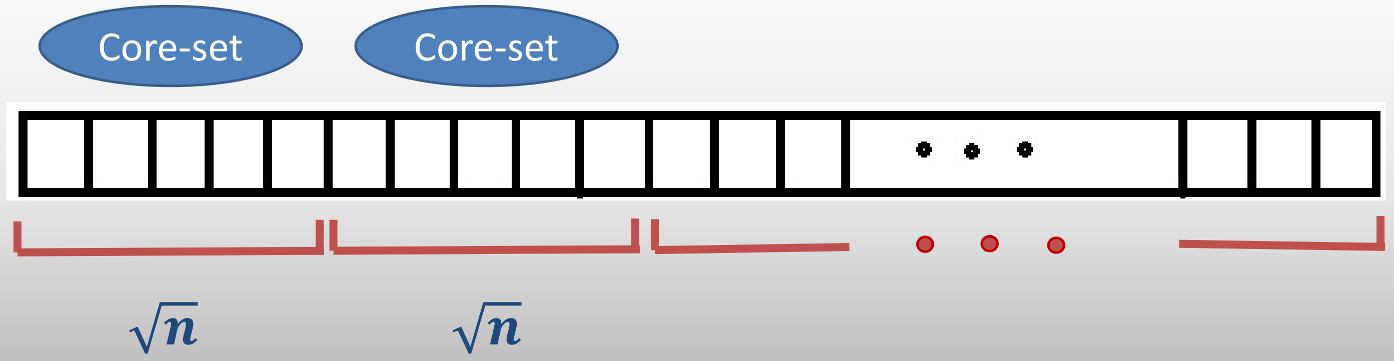

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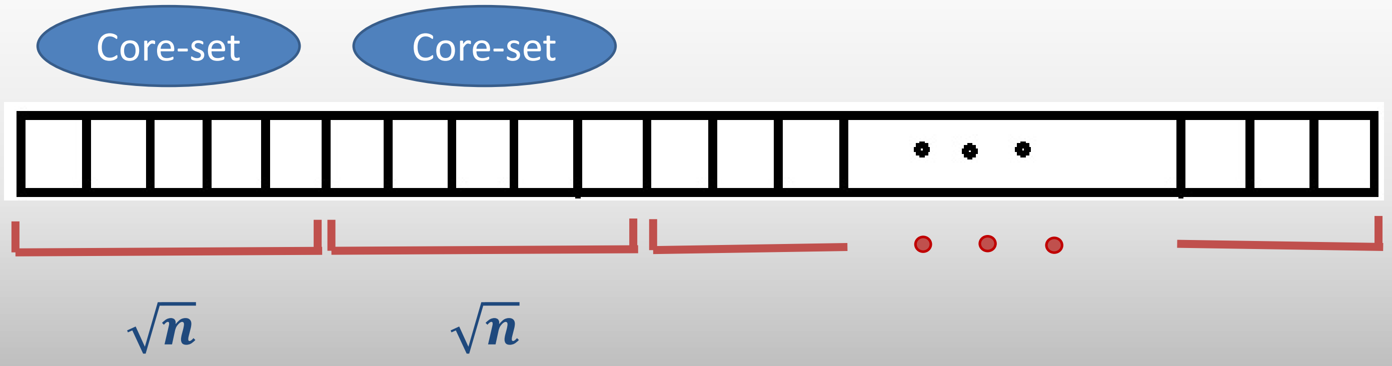
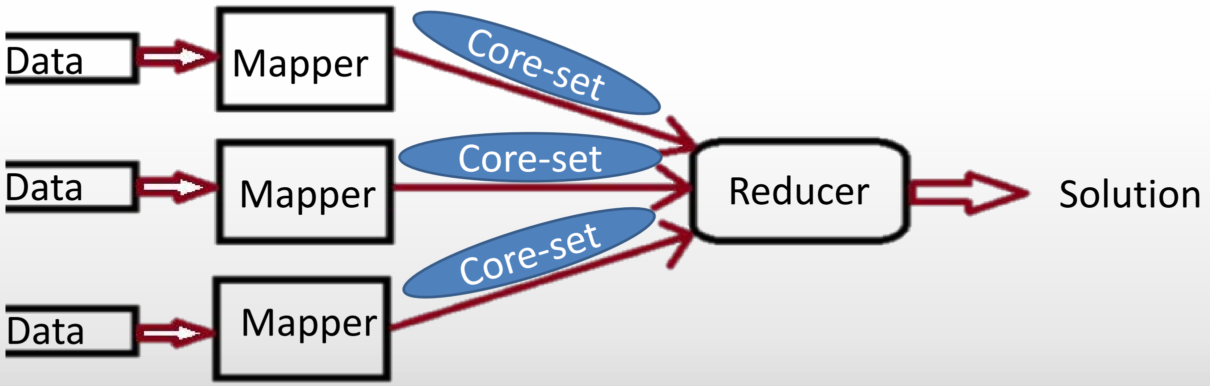
  - Space goes down from $n$ to $\sqrt{n}$



# Applications: Distributed Computation

- Streaming Computation
- **Distributed System:**
  - Each machine holds a block of data.
  - A composable core-set is computed and sent to the server



# Applications: Improving Runtime

- Streaming Computation
- Distributed System
- **Similar framework for improving the runtime**

Can we get a composable core-set of small size for the determinant maximization problem?

# Composable Core-sets for Volume Maximization

| | | [IMOR'18] |
|---|---|---|
| **Approximation** | | $\tilde{O}(k)^{k/2}$ |
| **Core-set Size** | | $\tilde{O}(k)$ |
| **Simple?** | | × |

**LP-based Optimal Approximation Algorithm of [IMOR'18]:**

There exists a polynomial time algorithm for computing an $\tilde{O}(k)^{k/2}$ -composable core-set of size $\tilde{O}(k)$ for the volume maximization problem.

# Composable Core-sets for Volume Maximization

| | Lower Bound | [IMOR'18] |
|---|---|---|
| **Approximation** | $\Omega(k)^{\frac{k}{2}-o(k)}$ | $\tilde{O}(k)^{\frac{k}{2}}$ |
| **Core-set Size** | $k^{O(1)}$ | $\tilde{O}(k)$ |
| **Simple?** | | × |

**Lower bound [IMOR'18]:**

Any composable core-set of size $k^{O(1)}$ for the volume maximization problem must have an approximation factor of $\Omega(k)^{\frac{k}{2}(1-o(1))}$.

# Our Results

| | Lower Bound | [IMOR'18] | Greedy |
|---|---|---|---|
| Approximation | $\Omega(k)^{\frac{k}{2}-o(k)}$ | $\widetilde{O}(k)^{\frac{k}{2}}$ | $O(C^{k^2})$ |
| Core-set Size | $k^{O(1)}$ | $\widetilde{O}(k)$ | $k$ |
| Simple? | | × | ✓ |

**The widely used Greedy algorithm** produces a composable core-set of size $k$ with approximation factor $O(C^{k^2})$.

# Our Results

| | Lower Bound | [IMOR'18] | Greedy | Local Search |
|---|---|---|---|---|
| Approximation | $\Omega(k)^{\frac{k}{2}-o(k)}$ | $\tilde{O}(k)^{\frac{k}{2}}$ | $O(C^{k^2})$ | $O(k^k)$ |
| Core-set Size | $k^{O(1)}$ | $\tilde{O}(k)$ | $k$ | $k$ |
| Simple? | | × | ✓ | ✓ |

**The Local Search Algorithm** produces a composable core-set of size $k$ with approximation factor $O(k)^{2k}$.

# This Talk

**The Local Search Algorithm** produces a composable core-set of size $k$ with approximation factor $O(k)^k$ for the volume maximization problem.

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**The Local Search Algorithm** produces a composable core-set of size $k$ with approximation factor $O(k)^k$ for the volume maximization problem.

**In comparison to the optimal core-set algorithm**

- Approximation $O(k)^{\boldsymbol{k}}$ as opposed to $O(k \log k)^{\boldsymbol{k/2}}$
- Smaller Size $k$ as opposed to $O(k \log k)$
- Simpler to implement (similar to Greedy)
- Better performance in practice

# The Local Search Algorithm

**Input:** a set $V$ of $n$ points and a parameter $k$

1. Start with an arbitrary subset of $k$ points $S \subseteq V$
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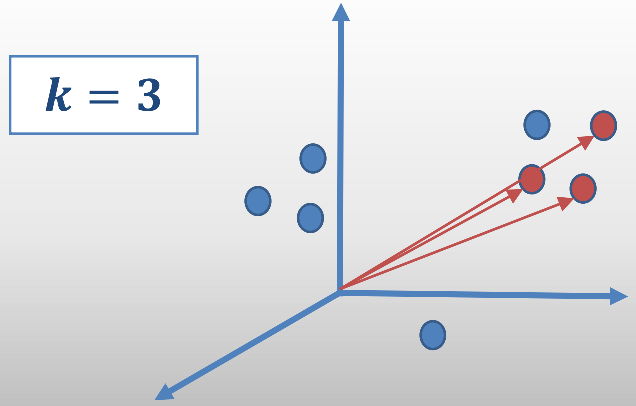

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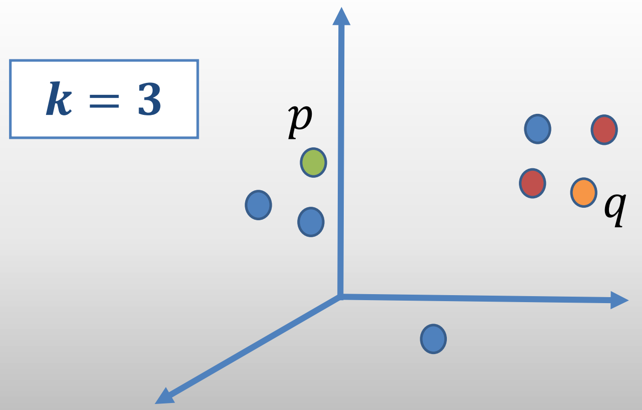

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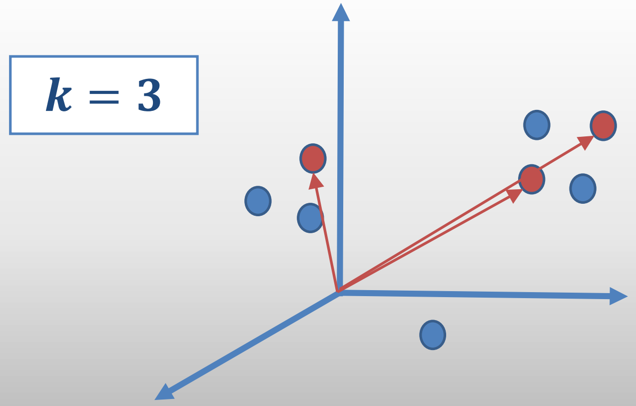

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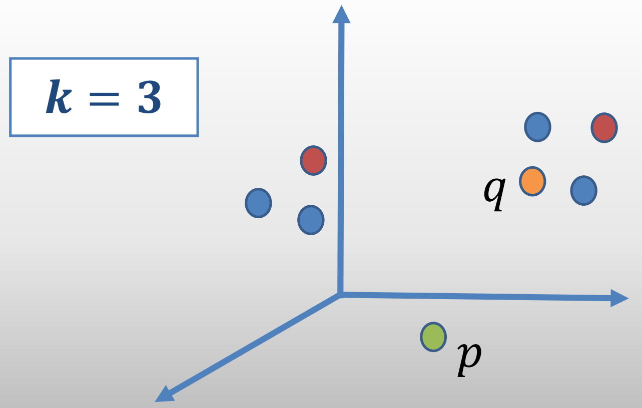

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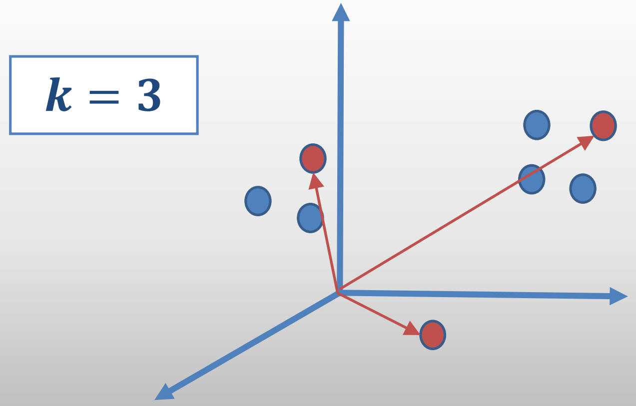

# To bound the run time

**Input:** a set $V$ of $n$ points a

Start with a crude approximation (Greedy algorithm)

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If it increases by at least a factor of $(1 + \epsilon)$

# Checking the condition

**Input:** a set $V$ of $n$ points and a parameter $k$

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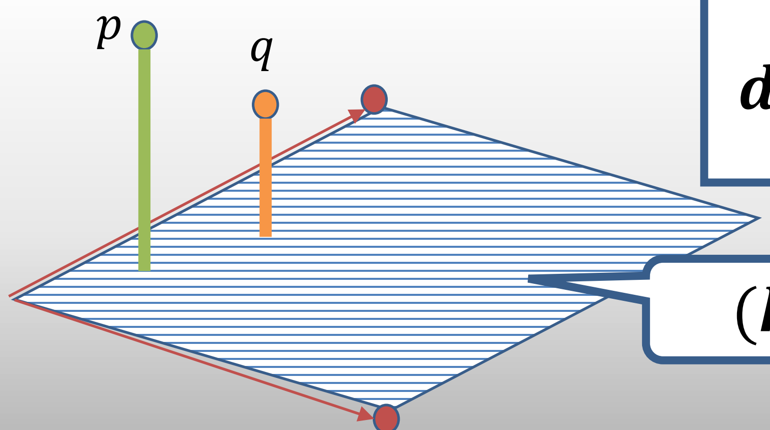
$$\boldsymbol{dist}(\boldsymbol{p}, \boldsymbol{H}_{\boldsymbol{S}\setminus\{\boldsymbol{q}\}}) > \boldsymbol{dist}(\boldsymbol{q}, \boldsymbol{H}_{\boldsymbol{S}\setminus\{\boldsymbol{q}\}})$$




**$(k-1)$-dimensional Subspace**

**Main Lemma [informal]:**

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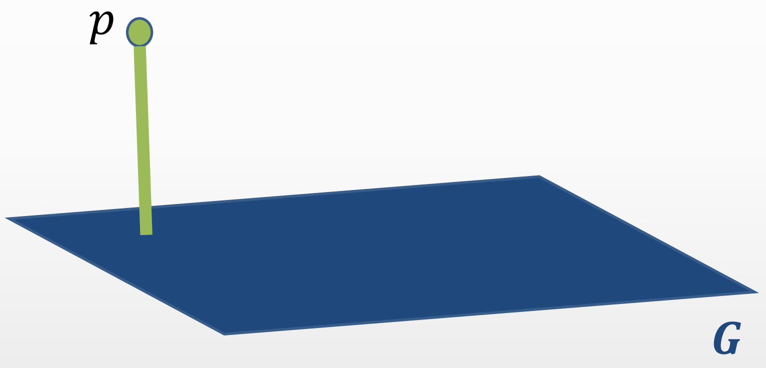

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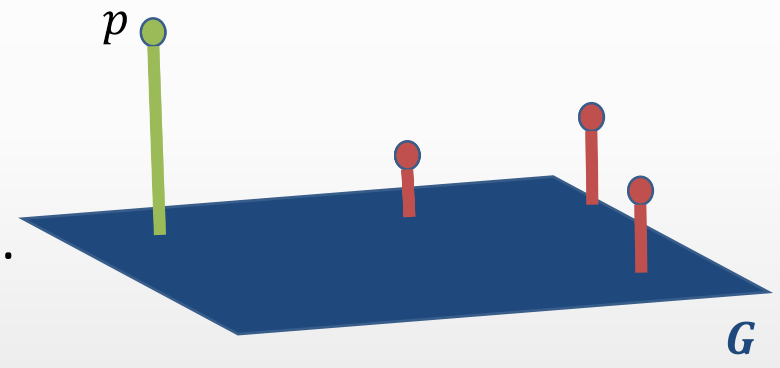

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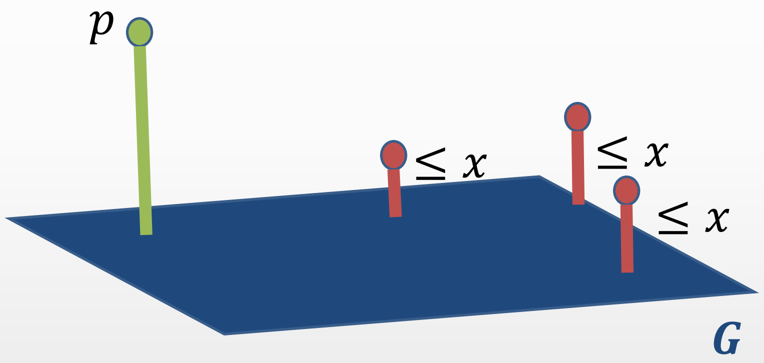

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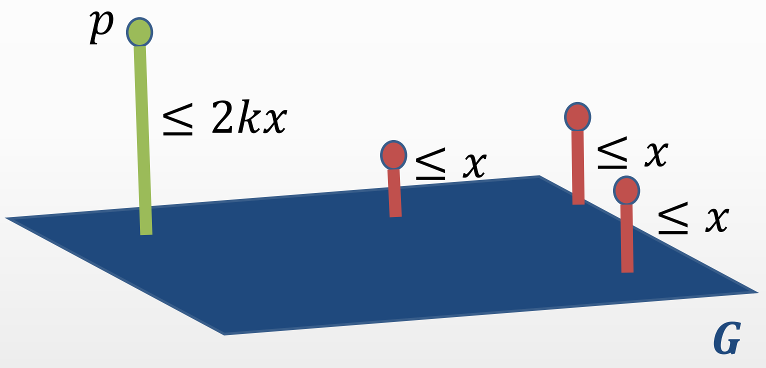


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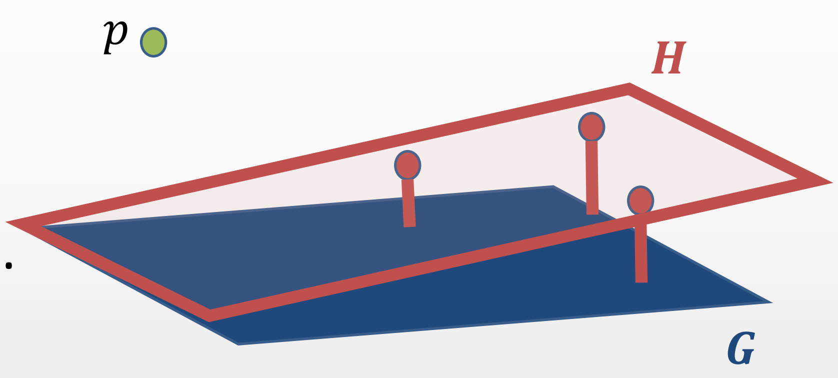


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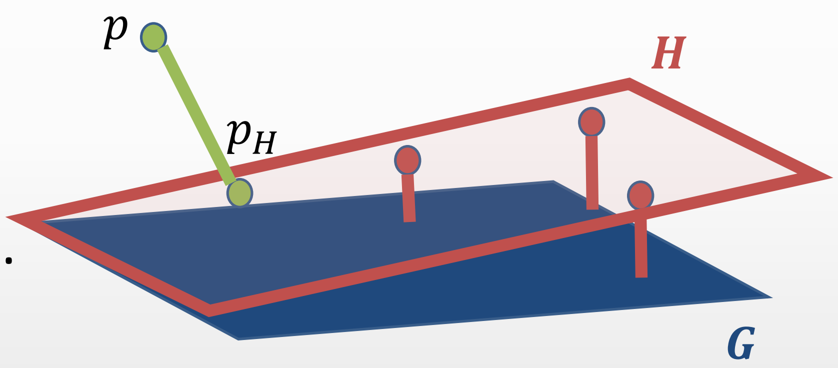


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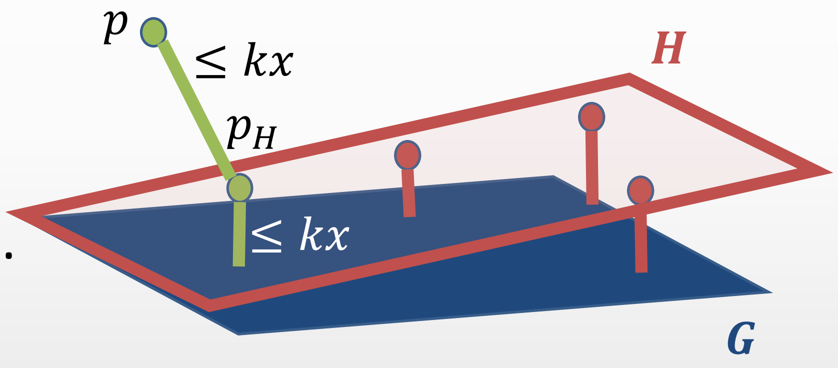


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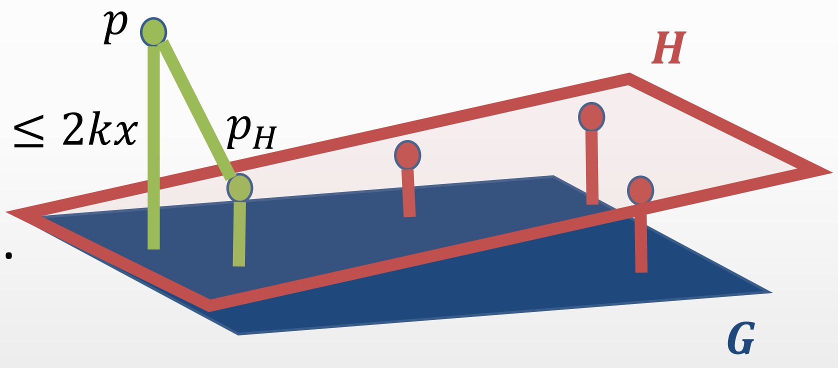


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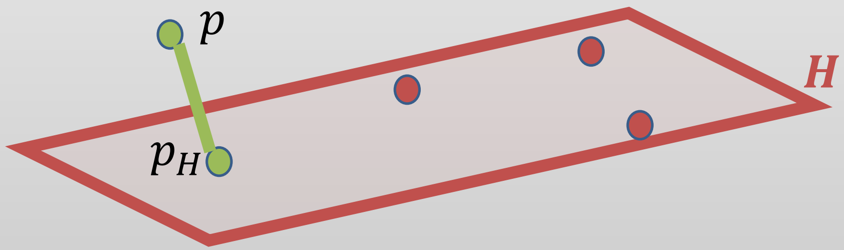

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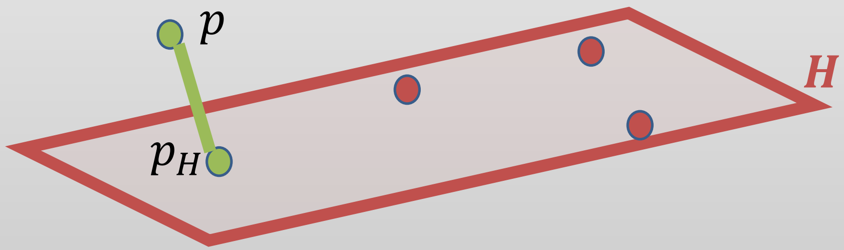

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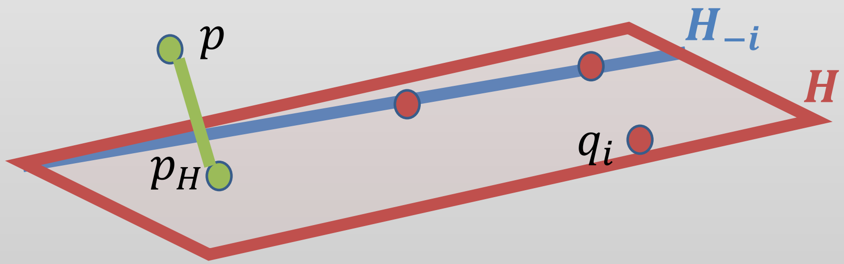

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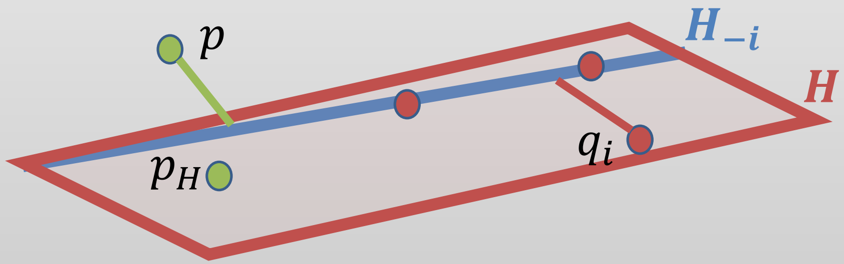

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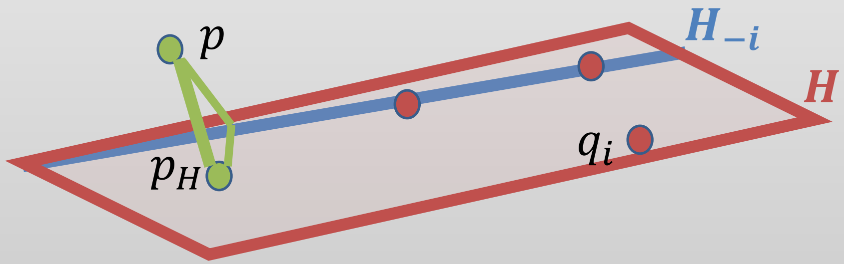

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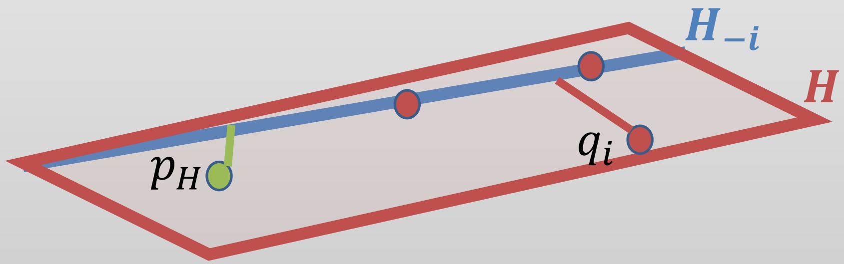

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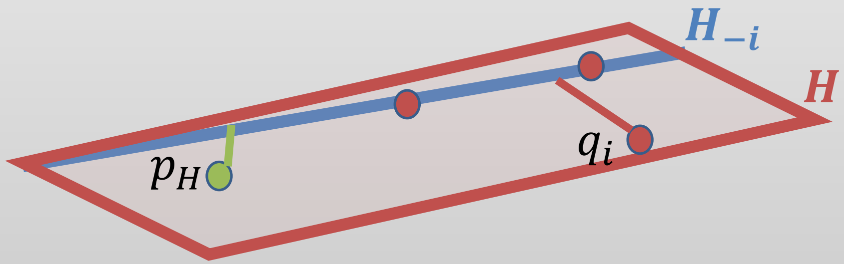

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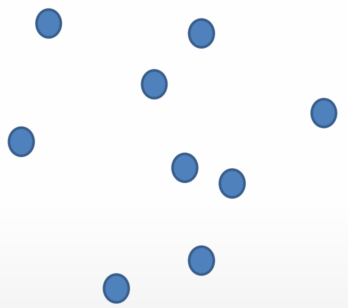
Local Search produces a core-set for volume maximization

Let $\boldsymbol{V} = \bigcup_i V_i$ be the union of the point sets



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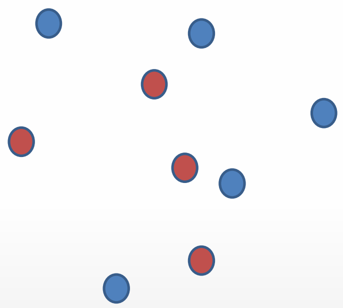

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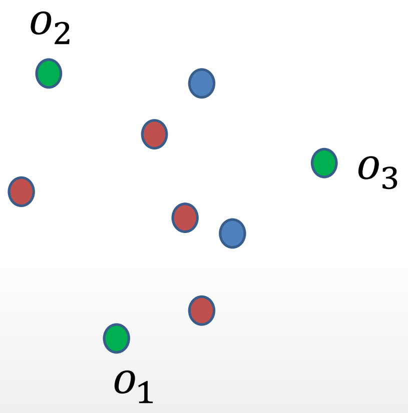

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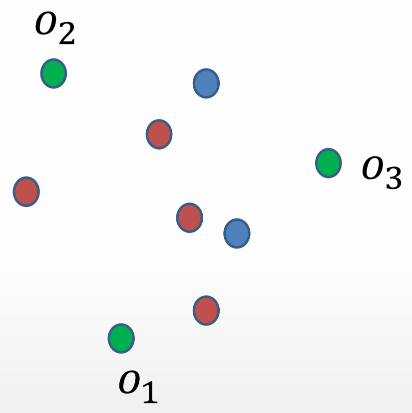

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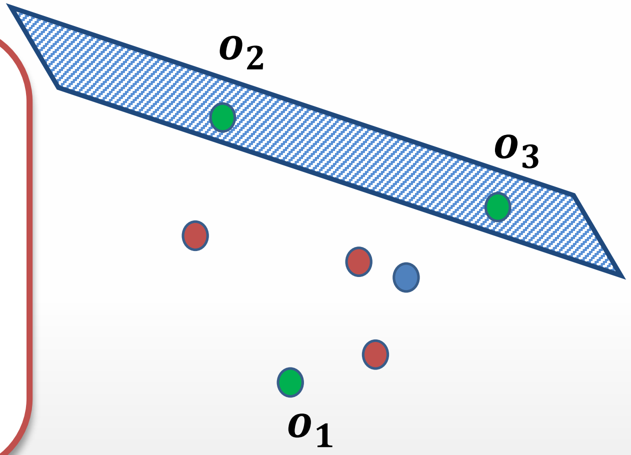

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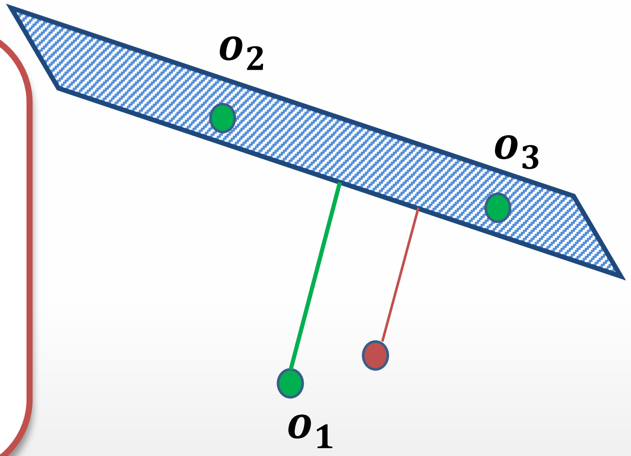

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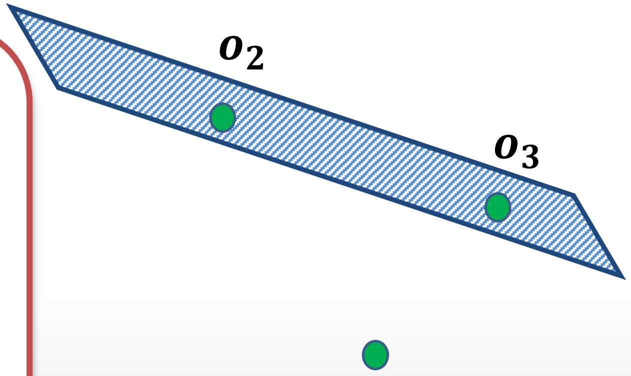

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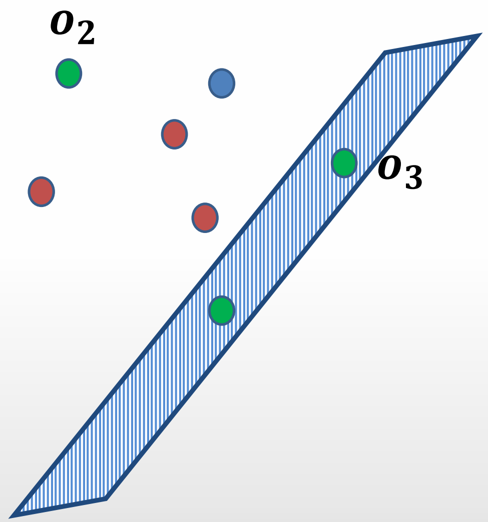

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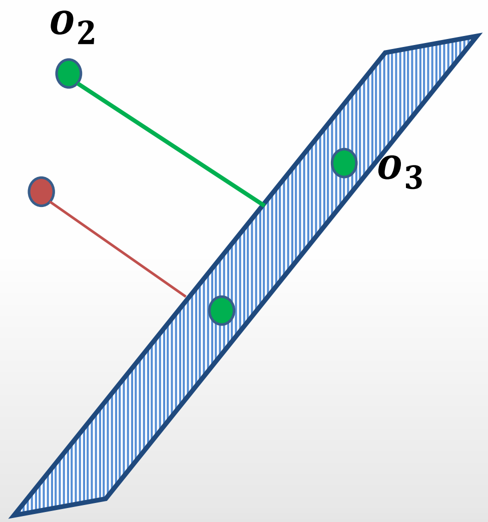

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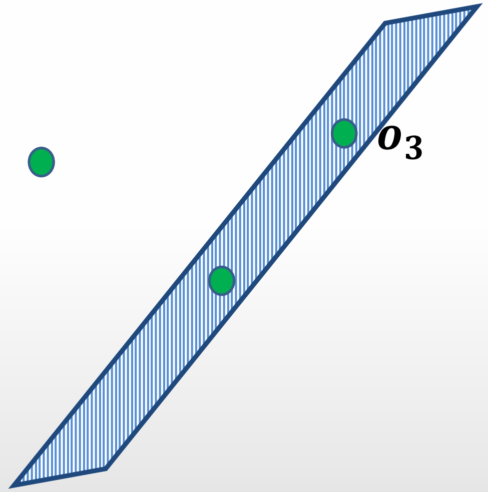

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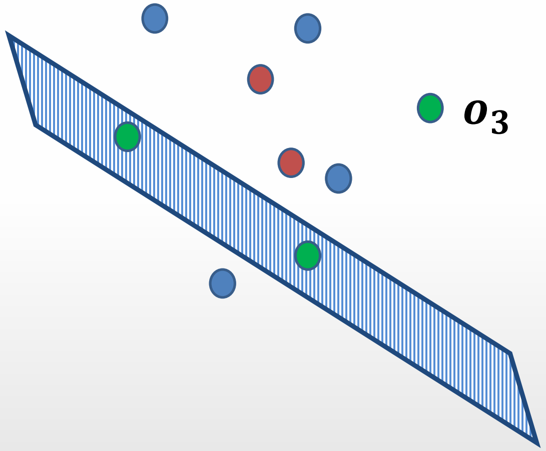

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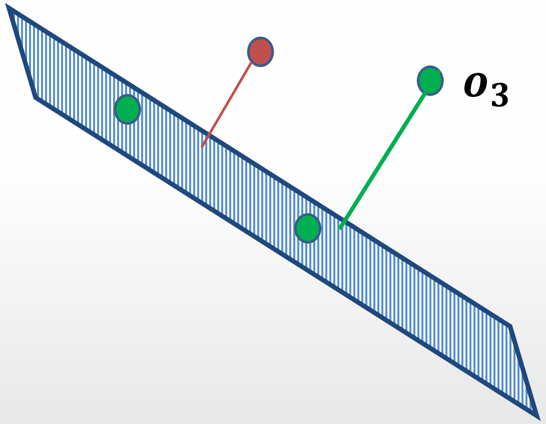

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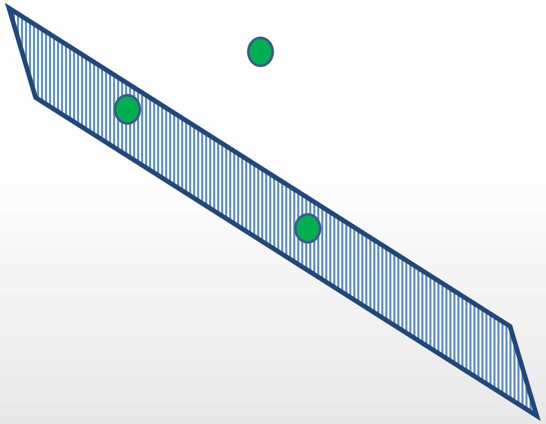

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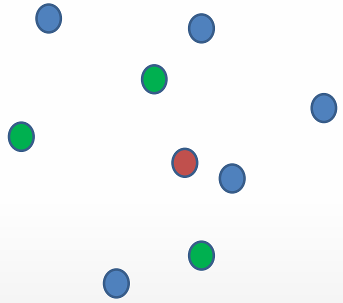

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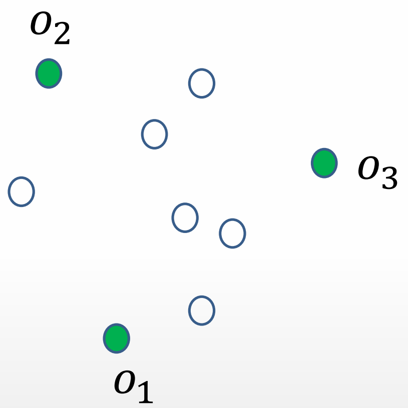

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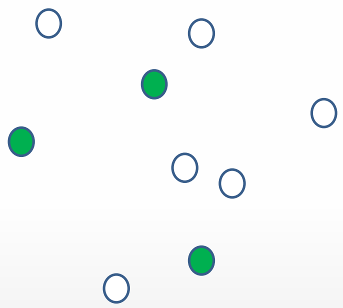

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Since local search preserve maximum distances to subspaces

➢ Lose a factor of at most $2k$ at each iteration

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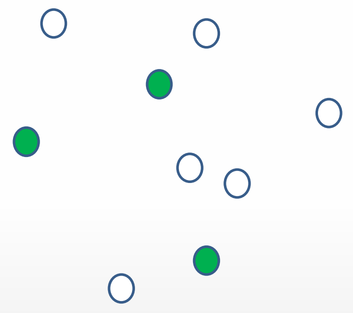


- Lose a factor of at most $2k$ at each iteration
- Total approximation factor $(2k)^k$

# Empirical Results

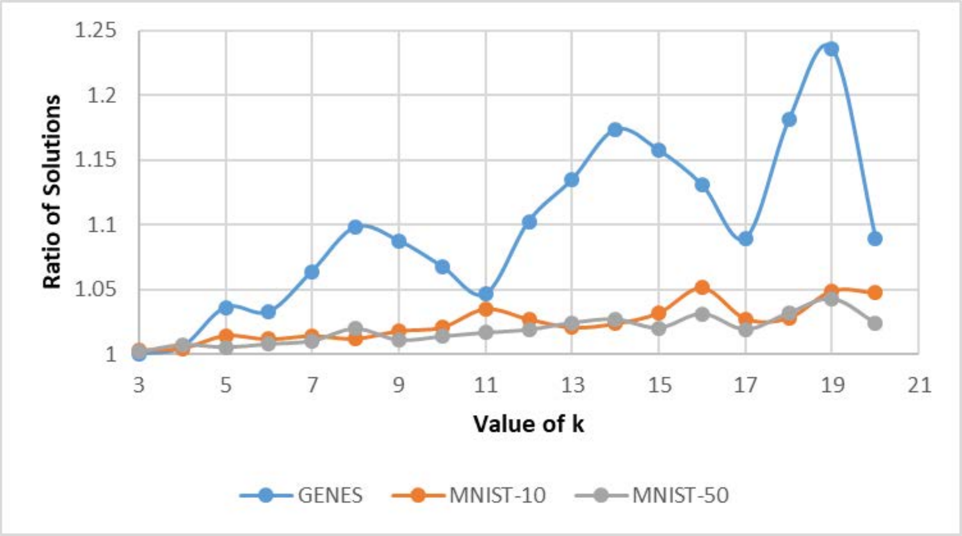
### Data Set

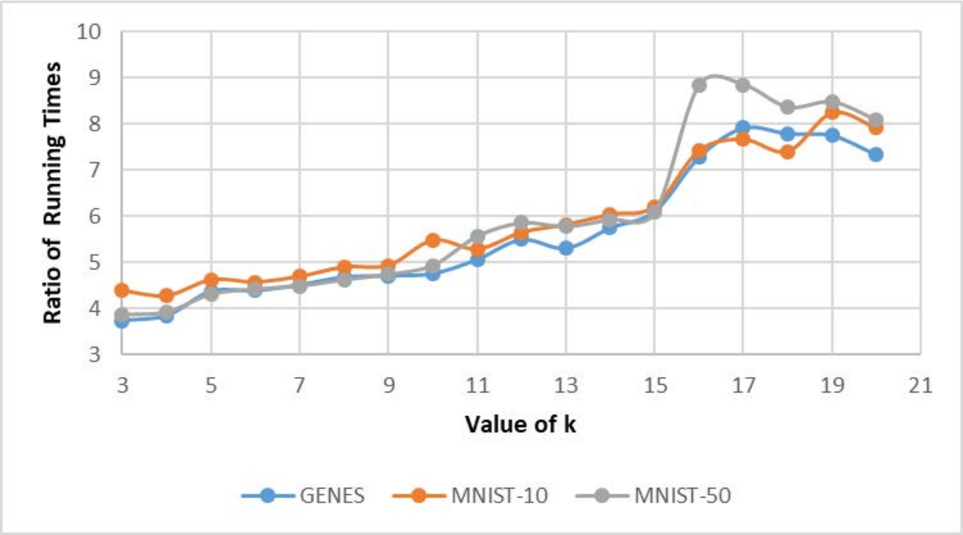
- MNIST, with number of parts = 10
- MNIST, with number of parts = 50
- GENES, with number of parts = 10

### Process

- Partition the data set randomly into parts
- Compute a core-set using one of the algorithms: **Greedy, Local Search, LP-Based algorithm of [IMOR'18]**
- Use greedy on the union of the coresets

# Local Search vs Greedy



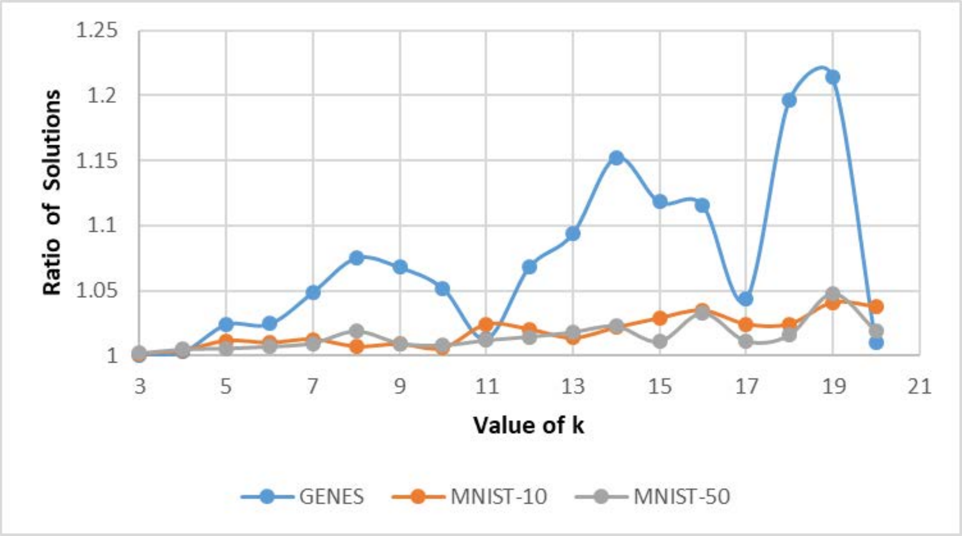

**Improvement of the solution of Local Search over Greedy**

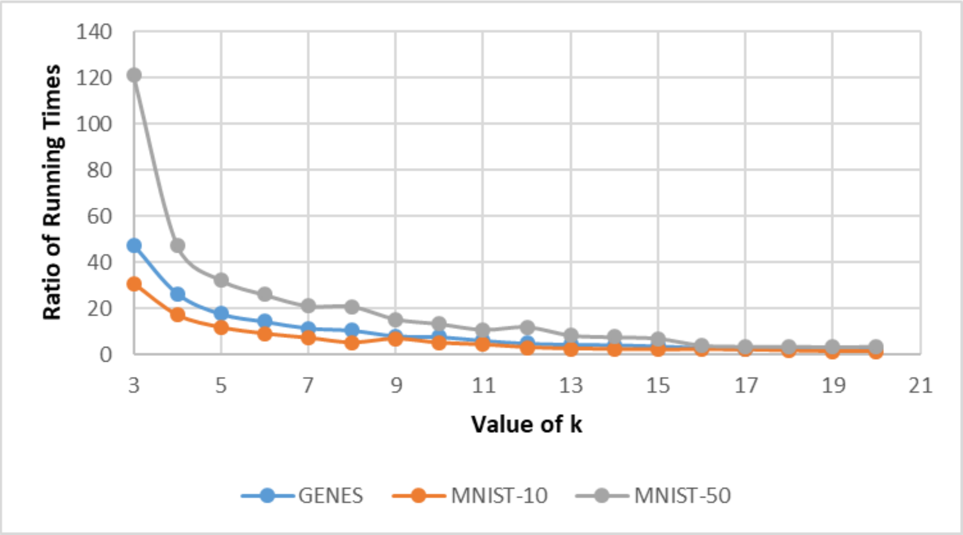
- On average, 1.2%, 2.5%, and 9.6% improvement
- Some cases up to 58% improvement

**Ratio of runtime of Local Search over Greedy**

- On average, 6 times slower

# Local Search vs. LP-based Algorithm of [IMOR'18]




**Improvement of the solution of Local Search over [IMOR'18]**

- On average, 1.4%, 1.8%, and 7.3% improvement
- Some cases up to 63% improvement

**Ratio of runtime of Local Search over [IMOR'18]**

- For lower values of k, Local Search is up to 50 times faster.

## Summary

- Volume/Determinant Maximization Problem
- Notion of composable core-sets
- Algorithms that find composable core-sets for volume/determinant maximization

| | [IMOR'18] | Greedy | Local Search |
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| Approximation | $O(k\log k)^{k/2}$ | $O(C^{k^2})$ | $O(k^k)$ |
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# THANK YOU!

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